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ASPECT RATIO OF SLENDER WEB & POSTBUCKLING BEHAVIOUR

POMER STRÁN ŠTÍHLEJ STENY A POKRITICKÉ PÔSOBENIE

Abstract

Postbuckling analysis of slender web loaded in compression is presented. The non-linear FEM equations [14] are derived from the variational principle of minimum of total potential energy [13]. To obtain the non-linear equilibrium paths, Newton-Raphson iteration algorithm [11], [12] is used. Peculiarities of the effect of the initial imperfections [7], [8] on load-deflection paths are investigated with respect to aspect ratio of the web. Special attention is focused on the postbuckling mode of the web.

Keywords

Stability, buckling & postbuckling, geometric nonlinear theory, initial imperfection, aspect ratio, Newton-Raphson method.

Abstrakt

V príspevku je analyzované pokritické pôsobenie tlačenej štíhlej steny. Systém nelineárnych podmienkových rovníc v metóde konečných prvkov [14] bol odvodený z variačného princípu minima celkovej potenciálnej energie [13]. Za účelom získania zaťažovacích dráh bola použitá Newton-Raphsonova iteračná metóda [11], [12]. Vplyv počiatočných tvarových imperfekcií [7], [8] na jednotlivé zaťažovacie dráhy pokritického pôsobenia bol skúmaný s ohľadom na pomer strán štíhlej steny.

Kľúčové slová

Stabilita, podkritické a pokritické pôsobenie, geometricky nelineárna teória, počiatočná imperfekcia, pomer strán, Newton-Raphsonova metóda.

1 INTRODUCTION

Solving stability of the slender web, it is often insufficient to determine the elastic critical load, i.e. the load, when ideal web starts buckling. It is necessary to include initial imperfections of real web into solution and determine limit load level more accurately. The geometrically non-linear theory represents a basis for the reliable description of the post-buckling behaviour of the slender web [2]. The result of the numerical solution represents high number of load versus displacement paths [10].

The mode of the buckling of the lowest elastic critical load is usually taken as the mode of the initial geometrical imperfections. In such a case we do not have the snap-through effects. To create the snap-through effect, the mode of the initial imperfections has to be taken as the combination of the mode of the lowest elastic critical load and the mode of the second elastic critical load [9].

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2 THEORY

We assume a rectangular slender web simply supported along the edges (Fig. 1) with the thickness t. The displacements of the point of the neutral surface are denoted $q = [u, v, w]^T$ and the related load vector is $p = [p_x, 0, 0]^T$. We assume the so called von Kármán theory, when the out of plane (plate) displacements (w) are much bigger as in-plane (web) displacements (u,v). Taking into account the non-linear terms we have the strains

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_{Lm} + \boldsymbol{\varepsilon}_{Nm} - z \cdot \boldsymbol{k} \,, \tag{1}$$

where $\boldsymbol{\varepsilon}_{Lm} = \left[u_{,x}, v_{,y}, u_{,y} + v_{,x}\right]^T$ – in plane (web) strains – linear part,

$\boldsymbol{\varepsilon}_{Nm} = \frac{1}{2}\left[w_{,x}^2, w_{,y}^2, 2w_{,x}.w_{,y}\right]^T$ – in plane (web) strains – nonlinear part,

$\boldsymbol{k} = \left[w_{,xx}, w_{,yy}, 2w_{,xy}\right]^T$ – out of plane (plate) part of the strains,

the indexes denote the partial derivations.

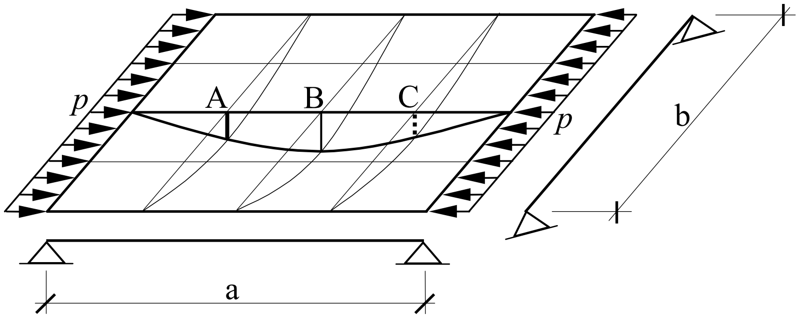

Fig. 1: Notations of the quantities of the slender web loaded in compression

The initial displacements will be assumed as the out of plane displacements only and so we have

$$\boldsymbol{\varepsilon}_0 = \boldsymbol{\varepsilon}_{0Nm} - z \cdot \boldsymbol{k} \,. \tag{2}$$

We assume the linear elastic material and the stresses can be written as:

$$\boldsymbol{\sigma} = \boldsymbol{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0), \text{ where } \boldsymbol{D} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}. \tag{3}$$

For such defined task, we can express the total potential energy as:

$$U = U_i + U_e = \int_V \frac{1}{2}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0)^T \boldsymbol{\sigma}\, dV - \int_A \boldsymbol{q}^T \boldsymbol{p} dA \tag{4}$$

Which leads after substituting and modification to:

$$U = \int_A \frac{1}{2}(\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0m})^T t \cdot \boldsymbol{D}(\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0m}) dA + \int_A \frac{1}{2}(\boldsymbol{k} - \boldsymbol{k}_0)^T \frac{t^3}{12}\boldsymbol{D}(\boldsymbol{k} - \boldsymbol{k}_0) dA - \int_A \boldsymbol{q}^T \boldsymbol{p}\, dA \tag{5}$$

The system of conditional equations we can get from the condition of the minimum of the increment of the total potential energy $\delta\,\Delta U = 0$. This system can be written as

$$\boldsymbol{K}_{inc}\,\Delta\boldsymbol{\alpha} + \boldsymbol{F}_{int} - \boldsymbol{F}_{ext} - \Delta\boldsymbol{F}_{ext} = 0\,, \tag{6}$$

where $\boldsymbol{K}_{inc}$ – the incremental stiffness matrix of the web,

$\boldsymbol{F}_{int}$ – the internal force of slender web,

$\boldsymbol{F}_{ext}$ – the external load of slender web,

$\Delta\boldsymbol{F}_{ext}$ – the increment of the external load of slender web.

To obtain the non-linear equilibrium paths, Newton-Raphson iteration algorithm is assumed. Equality of Jacobi matrix with the incremental stiffness matrix ($\boldsymbol{K}_{inc} \equiv \boldsymbol{J}$) is used and residues are evaluated as unbalanced nodal forces. The specific problem of using the FEM for the solution of non-linear problem of the post-buckling behaviour of the slender web is, that we do not compile the system of the algebraic equations, but even so we use the Newton-Raphson iteration with the combination of the incremental steps [11]. To be able to evaluate the different paths of the solution, the pivot term of the Newton-Raphson iteration has to be changed during the solution.

3 ILLUSTRATIVE EXAMPLES

Articles [4], [5] have summarized problems of postbuckling modes for slender web with aspect ratio $\alpha = 1$. Illustrative examples of steel web $a = b = 120$ *mm*, $t = 1$ *mm* are presented as load – displacement paths for different types of geometrical initial imperfections. In Figs. 2 and 3 we can see that two almost identical modes of initial imperfection at the beginning of the process offer two different solutions in postbuckling.

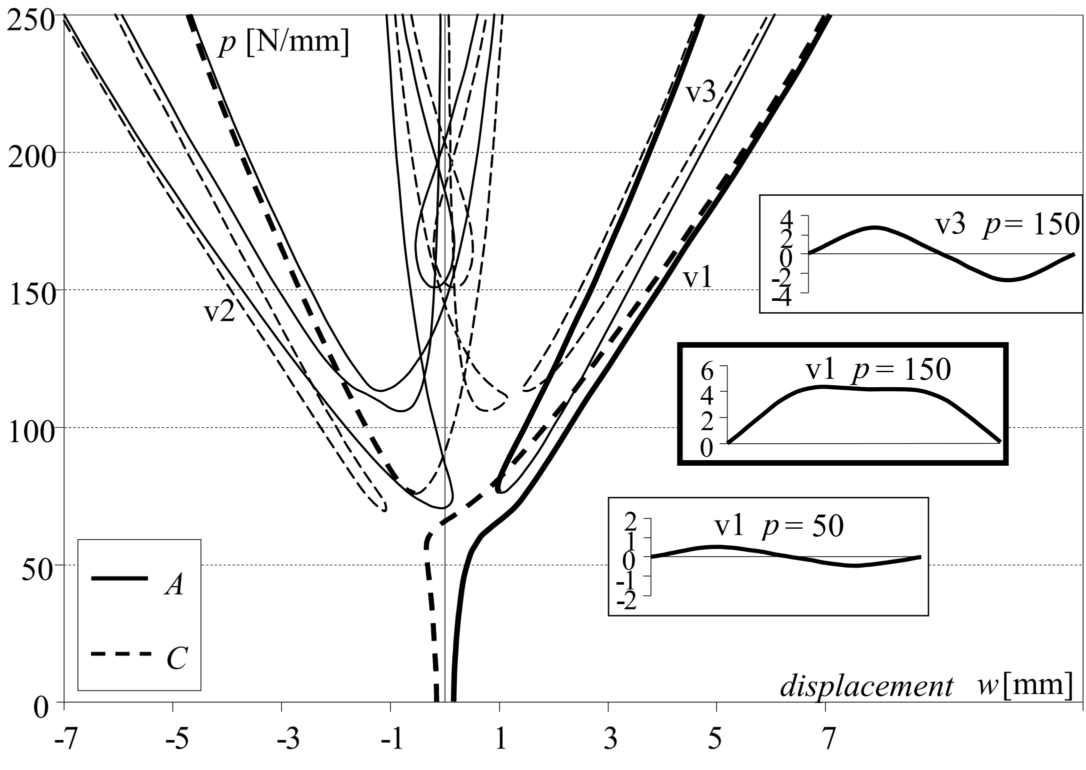

Fig. 2: The post-buckling behaviour of the slender web with the initial displacement f_{01}

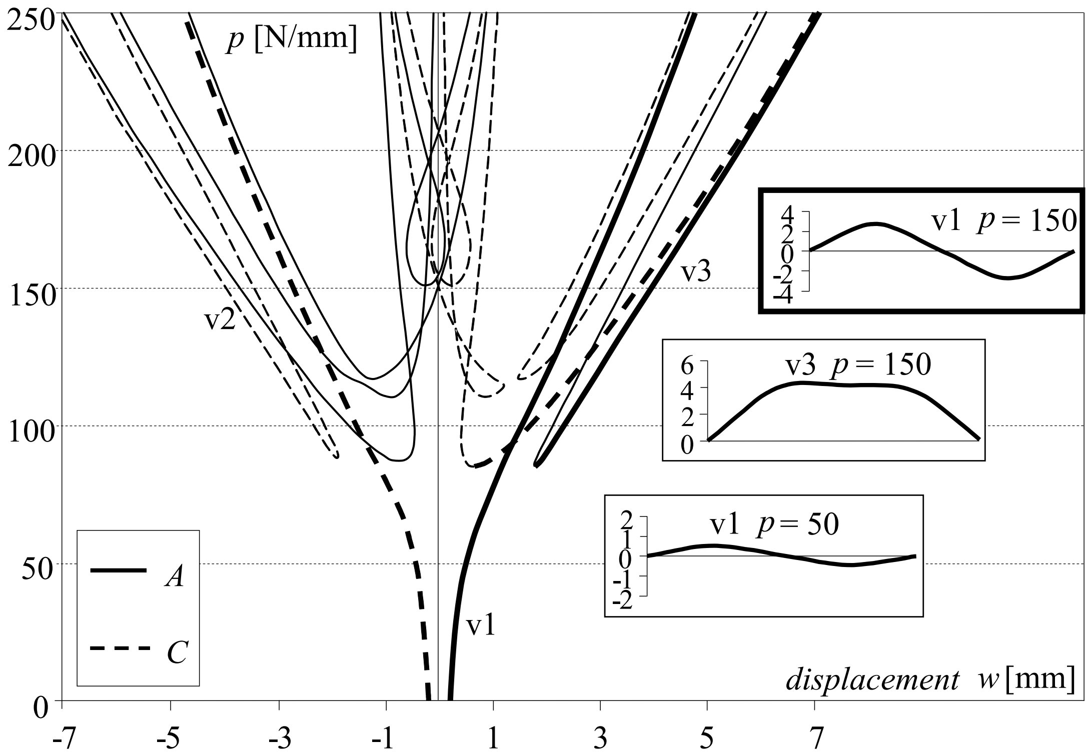

Fig. 3: The post-buckling behaviour of the slender web with the initial displacement f_{02}

Initial imperfections in presented solutions were considered as follows:

$$f_{01} = 0{,}01 \sin\frac{\pi \cdot x}{a} \sin\frac{\pi \cdot y}{b} + 0{,}15 \sin\frac{2\pi \cdot x}{a} \sin\frac{\pi \cdot y}{b} ,$$

$$f_{02} = 0{,}01 \sin\frac{\pi \cdot x}{a} \sin\frac{\pi \cdot y}{b} + 0{,}2 \sin\frac{2\pi \cdot x}{a} \sin\frac{\pi \cdot y}{b} .$$

Due to the mode of the initial imperfection, out of plane nodal displacements in *A* and *C* have been taken as the reference values (see Fig. 1). The thick lines represents the stable paths and the thin lines represents the unstable paths of the solution. More details about the solution of the equilibrium paths are mentioned in [7].

Solution of linearised stability problem (e.g. [1], [3]) of slender web considering different aspect ratios, illustrated in Fig. 4, is expressed in the form of a relationship between plate-buckling coefficient *K* and aspect ratio α . We can see, that slender webs buckle approximately into squares and have the same critical stress near $4{,}0\ \sigma_{EU}$ (critical elastic Euler stress).

This means, inter alia, the fact that the web with an aspect ratio $\alpha = 1$ and web with $\alpha = 2$ behave from the point of view of stability similarly.

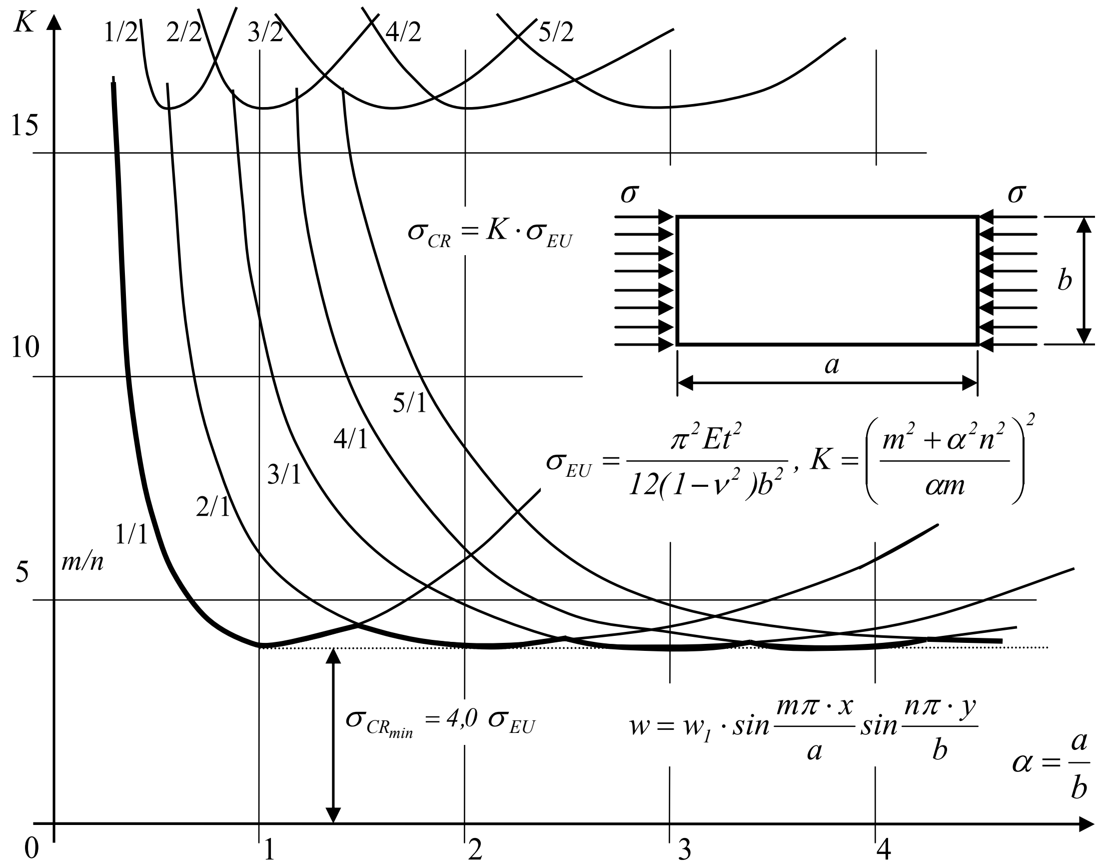

Fig. 4: Critical stress of slender web for different values of aspect ratio

Let us now analyze nonlinear solution of slender web with aspect ratio $\alpha = 2$. Example of steel web with $a = 240\ mm$, $b = 120\ mm$ and $t = 1\ mm$ is presented below (see also [6]). New load-displacement paths appear comparing to web with $\alpha = 1$ and solution has become unclear. Due to this reason, in Fig. 5, solution of ideal web without initial geometrical imperfections is presented first. Out of plane nodal displacements in *A* and *C* are plotted again, however paths are not separated in the diagram. In case of perfect web, some paths are doubled, the others overlap due to the symmetry of solution.

As expected, we see that paths representing buckling in mode 2-1 emerge from the bifurcation point of the lowest load level (#1). Load level in bifurcation point #2 (paths emerging from this bifurcation point represent buckling in mode 3-1) is also lower than the load level in bifurcation point #3 with paths representing buckling in basic mode 1-1 (which fully corresponds to the results in Fig. 4). Also paths turning in limit points #3* represent buckling in mode 1-1 (with higher corresponding load level). Other paths shown around bifurcation point #2 represent different forms of mode transformation between modes 2-1 and 3-1. Sections through the buckling area on mentioned equilibrium paths are also depicted in Fig. 5.

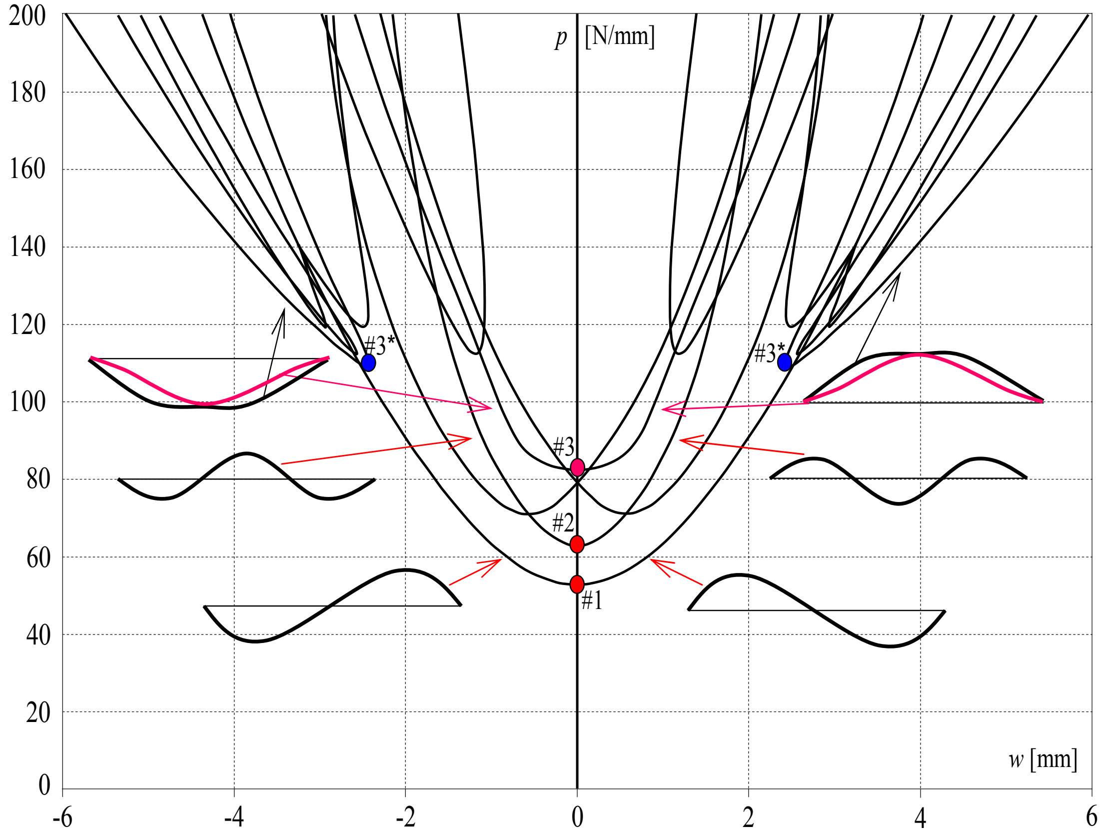

Fig. 5: Web without imperfections, aspect ratio $\alpha = 2$

Nonlinear solution of slender web with initial geometrical imperfection similar to mode 1-1 is presented in Fig. 6. Thick continuous lines denote the displacement in *A*, thick dashed lines refer to displacement in *C*. Solution of perfect web without initial imperfections is plotted by thin dashed lines on the background for comparison. We can see how individual paths of imperfect solution have been placed, when mode of buckling is similar to solution of perfect web and vice versa, when is significantly different. To simplify the figure, only the paths being mentioned in the text are plotted here.

Web buckles in mode 1-1 on the fundamental path of solution (identical to the mode of initial imperfection). When the load reaches a value close to the load level in bifurcation point #1 of perfect web solution, line representing displacement in *C* rapidly moves into negative values and postbuckling mode of the web is 2-1.

The solution marked No. 2 also represents the mode of buckling 2-1. In this case displacement in *A* reaches negative values. Due to the nearly symmetrical shape of initial imperfection stable paths of this are located close to the solution No. 1 (the difference between values of total potential energy of these two solutions is minimal).

As mentioned above, solutions marked No. 3 and 4 represent different forms of mode transformation between 2-1 and 3-1. There is naturally greater difference in load level at limit points of these solutions.

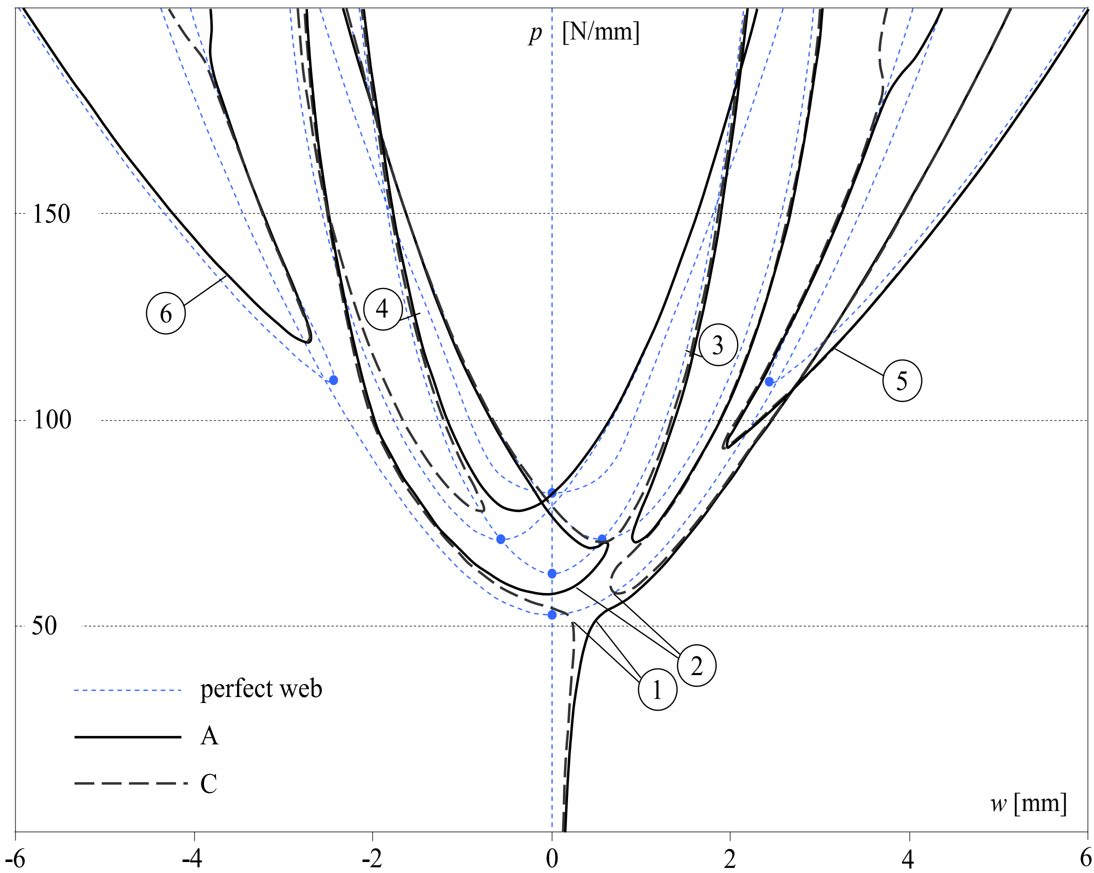

Fig. 6: Web with initial imperfection $f_{03} = 0{,}2\,sin\dfrac{\pi \cdot x}{a}\,sin\dfrac{\pi \cdot y}{b} + 0{,}01\,sin\dfrac{2\pi \cdot x}{a}\,sin\dfrac{\pi \cdot y}{b}$

The most noticeable difference is that in case of solutions marked No. 5 and 6. Solution No. 5 represents buckling in mode 1-1, with buckling on the side of initial imperfection. The load level in limit point is greatly reduced comparing to limit point level of perfect flat web (thin dashed line). In this case the total potential energy is reduced by the value representing the energy required for deformation of the perfect web to form of initial imperfection. Conversely, the solution marked No. 6, also mode 1-1, represents buckling on the opposite side of web as the initial imperfection is. The load level in this limit point is significantly higher comparing to limit point level of perfect flat web. In this case, the energy barrier required to reset the initial imperfections (perfect web) must be eliminated first, resulting in the increase of the load in this limit point.

4 CONCLUSION

The influence of the mode of the initial geometrical imperfections for the postbuckling of the slender web is presented with respect to aspect ratio of this web. The result representing a lot of load versus displacement paths is analyzed. A comprehensive analysis of the obtained solutions will be possible after supplementing the results of the value of total potential energy (as it was presented for square web in [4]). This supplement will be incorporated in the near future.

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